Exercise 29

Find the derivative of the function.

$$H(r) = \frac{(r^2-1)^3}{(2r+1)^5}$$

Solution

Take the derivative using the quotient rule and the chain rule.

$$\begin{aligned}
H'(r) = \frac{dH}{dr} &= \frac{d}{dr}\left[\frac{(r^2-1)^3}{(2r+1)^5}\right] \\
&= \frac{\left[\frac{d}{dr}(r^2-1)^3\right](2r+1)^5 - \left[\frac{d}{dr}(2r+1)^5\right](r^2-1)^3}{(2r+1)^{10}} \\
&= \frac{\left[3(r^2-1)^2 \cdot \frac{d}{dr}(r^2-1)\right](2r+1)^5 - \left[5(2r+1)^4 \cdot \frac{d}{dr}(2r+1)\right](r^2-1)^3}{(2r+1)^{10}} \\
&= \frac{\left[3(r^2-1)^2 \cdot (2r)\right](2r+1)^5 - \left[5(2r+1)^4 \cdot (2)\right](r^2-1)^3}{(2r+1)^{10}} \\
&= \frac{6r(r^2-1)^2(2r+1)^5 - 10(2r+1)^4(r^2-1)^3}{(2r+1)^{10}} \\
&= \frac{2(r^2-1)^2(2r+1)^4[3r(2r+1) - 5(r^2-1)]}{(2r+1)^{10}} \\
&= \frac{2(r^2-1)^2(2r+1)^4(r^2+3r+5)}{(2r+1)^{10}} \\
&= \frac{2(r^2-1)^2(r^2+3r+5)}{(2r+1)^6}
\end{aligned}$$